MATRICES WORKSHEET

1. The 2×2 matrix C is defined, in terms of a scalar constant k, by $C = \begin{pmatrix} 3 & k \\ 6 & 4 \end{pmatrix}$. Determine the value of k, given that the matrix C is singular.

2. Given the matrix $W = \begin{pmatrix} 3 & 6 \\ -2 & 5 \end{pmatrix}$, determine

   (i) the 2×2 matrix L such that $W + L = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

   (ii) the 2×2 matrix P such that $WP = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

3. Calculate the value of x and the value of y in the matrix equation below. $\begin{pmatrix} 1 & 5 \\ 2 & y \end{pmatrix} \begin{pmatrix} -4 & 1 \\ 2 & 9 \end{pmatrix} = \begin{pmatrix} x & 46 \\ 6 & 65 \end{pmatrix}$

4. Three matrices are given as follows: $P = \begin{pmatrix} -1 & 2 \\ 0 & 5 \end{pmatrix}$, $Q = \begin{pmatrix} a \\ b \end{pmatrix}$ and $R = \begin{pmatrix} 11 \\ 15 \end{pmatrix}$

   (i) Using a calculation to support your answer, explain whether matrix P is a singular or a non-singular matrix.

   (ii) Given that $PQ = R$, determine the values of a and b.

   (iii) State the reason why the matrix product QP is not possible.

5. The matrices A and B are given as $A = \begin{pmatrix} -1 & 0 \\ 3 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} -3 & 2 \\ 1 & 1 \\ 4 & 6 \end{pmatrix}$.

   (i) Determine A^{-1}, the inverse of A.

   (ii) Show that $A^{-1}A = I$, the identity matrix.

   (iii) Determine the matrix A^2.

   (iv) (a) Explain why the matrix product AB is NOT possible.

   (b) Without calculating, state the order of the matrix product BA.

6. The matrix M is defined as $M = \begin{pmatrix} 2p & -3 \\ 4 & 1 \end{pmatrix}$. Determine the value of p for which the matrix M is singular.

7. A and B re two 2×2 matrices such that $A = \begin{pmatrix} 1 & 2 \\ -4 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & -1 \\ 0 & 3 \end{pmatrix}$.

   (i) Calculate $2A + B$.

   (ii) Determine B^{-1}, the inverse of B.

(iii) Given that $\begin{pmatrix} 5 & -1 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ 3 \end{pmatrix}$, calculate the values of x and y.

8. L and M are two matrices where $L = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$ and $M = \begin{pmatrix} -1 & 3 \\ 0 & 2 \end{pmatrix}$. Evaluate

   (i) $L + 2M$ (ii) LM

9. The matrix, Q, is such that $Q = \begin{pmatrix} 4 & 2 \\ 1 & 1 \end{pmatrix}$

   (i) Find Q^{-1}

   (ii) Using a matrix method, find the values of x and y in the equation $\begin{pmatrix} 4 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 3 \end{pmatrix}$

10. L and N are two matrices where $L = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$ and $N = \begin{pmatrix} -1 & 3 \\ 0 & 2 \end{pmatrix}$. Evaluate $L - N^2$.

11. The matrix, M, is given as $M = \begin{pmatrix} x & 12 \\ 3 & x \end{pmatrix}$. Calculate the values of x for which M is singular.

12. Calculate the matrix product $3AB$, where $A = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}$.

13. A, B and C are matrices such that: $A = \begin{pmatrix} 2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & x \\ y & -2 \end{pmatrix}$ and $C = \begin{pmatrix} 5 & 6 \end{pmatrix}$. Given that $AB = C$,

    calculate the values of x and y.

14. Given that $R = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$

    (i) Show that R is non-singular.

    (ii) Find R^{-1}, the inverse of R

    (iii) Show that $RR^{-1} = I$

    (iv) Using a matrix method, solve the pair of simultaneous equation

$$2x - y = 0$$

$$x + 3y = 7$$

ANSWERS

1. $k = 2$

2. (i) $L = \begin{pmatrix} -3 & -6 \\ 2 & -5 \end{pmatrix}$ (ii) $P = \frac{1}{27}\begin{pmatrix} 5 & -6 \\ 2 & 3 \end{pmatrix}$

3. $x = 6, y = 7$

4. (i) Non-singular (ii) $a = -5, b = 3$ (iii) Number of columns in $Q \neq$ Number or rows in P

5. (i) $\frac{1}{-2}\begin{pmatrix} 2 & 0 \\ -3 & -1 \end{pmatrix}$ (ii) (iii) $\begin{pmatrix} 1 & 0 \\ 3 & 4 \end{pmatrix}$ (iv) (a) Number of columns in $A \neq$ Number or rows in B

   (b) 3×2

6. $p = -6$

7. (i) $\begin{pmatrix} 7 & 3 \\ -8 & 9 \end{pmatrix}$ (ii) $\frac{1}{15}\begin{pmatrix} 3 & 1 \\ 0 & 5 \end{pmatrix}$ (iii) $x = 2, y = 1$

8. (i) $\begin{pmatrix} 1 & 8 \\ 1 & 8 \end{pmatrix}$ (ii) $\begin{pmatrix} -3 & 13 \\ -1 & 11 \end{pmatrix}$

9. (i) $\frac{1}{2}\begin{pmatrix} 1 & -2 \\ -1 & 4 \end{pmatrix}$ (ii) $x = 1, y = 2$

10. $\begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$

11. $x = \pm 6$

12. $\begin{pmatrix} 21 & 54 \\ 15 & 42 \end{pmatrix}$

13. $x = 4, y = 3$

14. (i) $|R| = 7$ (ii) $\frac{1}{7}\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ (iii) (iv) $x = 1, y = 2$